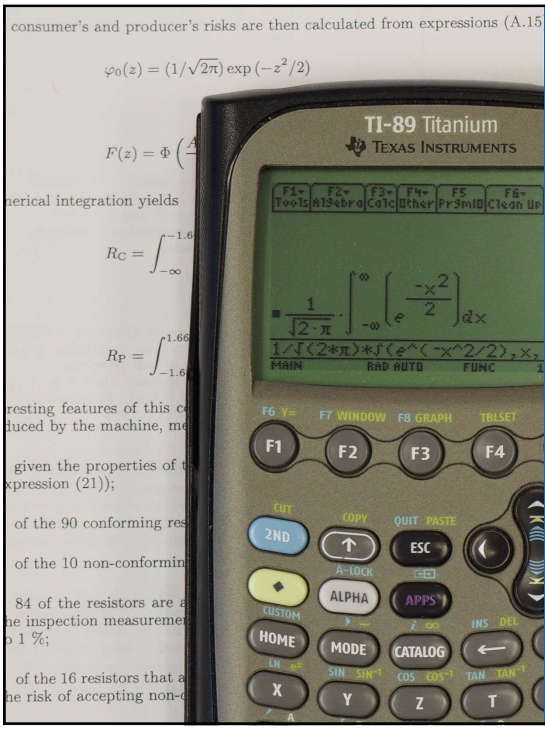


Discussion on Decision Rule and Statement of Conformity

Nov 2019

Standards and Calibration Laboratory

Why Do We Need to Discuss Statement of Conformity and Decision Rule ?

Reporting of Compliance

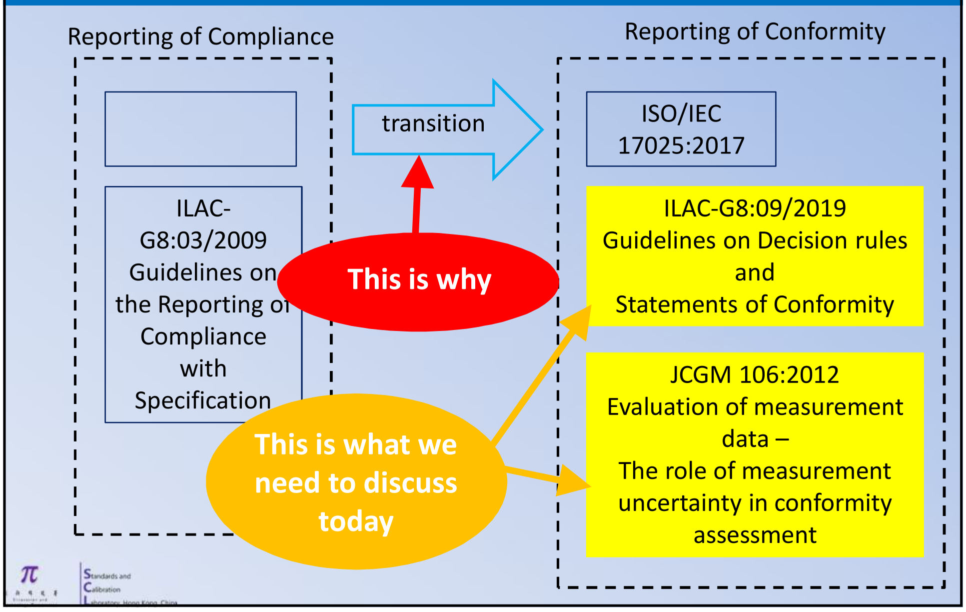
- ISO/IEC 17025:2005
- ILAC-G8:03/2009 Guidelines on the Reporting of Compliance with Specification

transition →

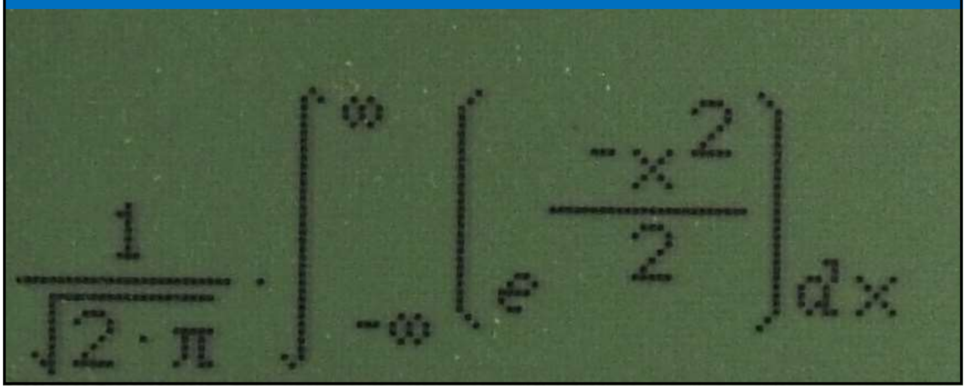
Reporting of Conformity

- ISO/IEC 17025:2017
- ILAC-G8:09/2019 Guidelines on Decision rules and Statements of Conformity
- JCGM 106:2012 Evaluation of measurement data – The role of measurement uncertainty in conformity assessment

Why Do We Need to Discuss Statement of Conformity and Decision Rule ?



ISO/IEC 17025:2005 on Statement of Compliance



ISO/IEC 17025:2005 on Statement of Compliance

5.10.3.1 In addition to the requirements listed in 5.10.2, test reports shall, where necessary for the interpretation of the test results, include the following:

…

b) where relevant, a statement of compliance/non-compliance with the requirements and/or specifications;

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From ISO/IEC 17025:2005

ISO/IEC 17025:2005 on Statement of Compliance

5.10.4.2 The calibration certificate shall relate only to quantities and the results of functional tests. If a statement of compliance with a specification is made, this shall identify which clauses of the specification are met or not met.

When a statement of compliance with a specification is made omitting the measurement results and associated uncertainties, the laboratory shall record those results and maintain them for possible future reference.

When statements of compliance are made, the uncertainty of measurement shall be taken into account.

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From ISO/IEC 17025:2005

ISO/IEC 17025:~~2005~~2017
on Statement of ~~Compliance~~Conformity and Decision Rule

$$\frac{1}{\sqrt{2\cdot\pi}}\cdot\int_{-\infty}^{\infty}\left(e^{\frac{-x^2}{2}}\right)dx$$

ISO/IEC 17025:2017
on Statement of Conformity and Decision Rule

3.7 **decision rule**

rule that describes how measurement uncertainty is accounted for when stating conformity with a specified requirement.

From ISO/IEC 17025:2017

ISO/IEC 17025:2017
on Statement of Conformity and Decision Rule

6.2.6 The laboratory shall authorize personnel to perform specific laboratory activities, including but not limited to, the following:

…

b) Analysis of results, including statements of conformity or opinions and interpretations;

7.1.3 When the customer requests a statement of conformity to a specification or standard for the test or calibration (e.g. pass/fail, in-tolerance/out-of-tolerance), the specification or standard and the decision rule shall be clearly defined. Unless inherent in the requested specification or standard, the decision rule selected should be communicated to, and agreed with, the customer.

From ISO/IEC 17025:2017

ISO/IEC 17025:2017
on Statement of Conformity and Decision Rule

7.8.3.1 In addition to the requirements listed in 7.8.2, test reports shall, where necessary for the interpretation of the test results, include the following:

…

(b) where relevant, a statement of conformity with requirements or specifications (see 7.8.6);

7.8.4.1 In addition to the requirements listed in 7.8.2, calibration certificates shall include the following:

…

(e) where relevant, a statement of conformity with requirements or specifications (see 7.8.6);

From ISO/IEC 17025:2017

ISO/IEC 17025:2017 on Statement of Conformity and Decision Rule

7.8.6 **Reporting statements of conformity**

7.8.6.1 When a statement of conformity to a specification or standard is provided, the laboratory shall document the decision rule employed, taking into account the level of risk …

(Note: where the decision rule is prescribed by the customer, regulations or normative documents, a further consideration of the level of risk is not necessary.)

7.8.6.2 The laboratory shall report on the statement of conformity, such that the statement clearly identified:

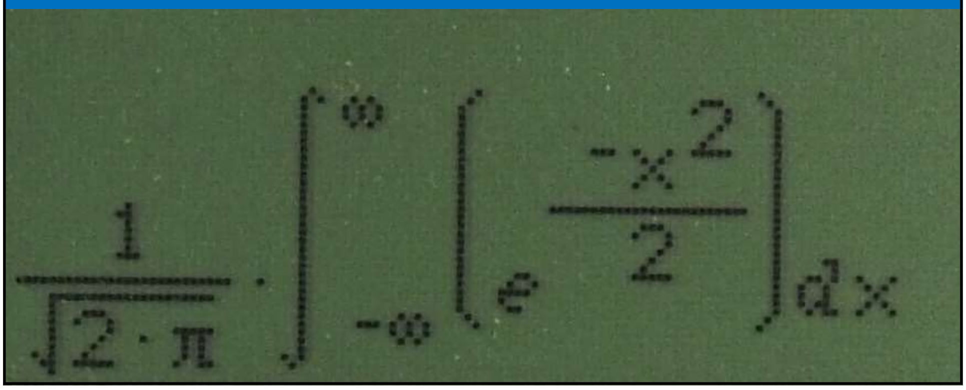
a) to which results the statement of conformity applies;

b) which specifications, standards or parts thereof are met or not met;

c) the decision rule applied (unless it is inherent in the requested specification or standard).

From ISO/IEC 17025:2017

ILAC-G8:03/2009 Guidelines on the Reporting of Compliance with Specification

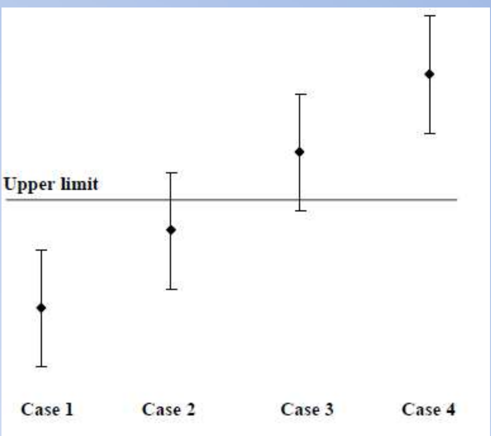


ILAC-G8:03/2009
on Statement of Compliance

- For specifications with upper and lower limits, a statement of compliance or non-compliance should only be made when the **expanded uncertainty** interval is reasonably small compared to the specified interval.
- The **coverage probability** for the expanded uncertainty should be specified.

ILAC-G8:03/2009
on Statement of Compliance

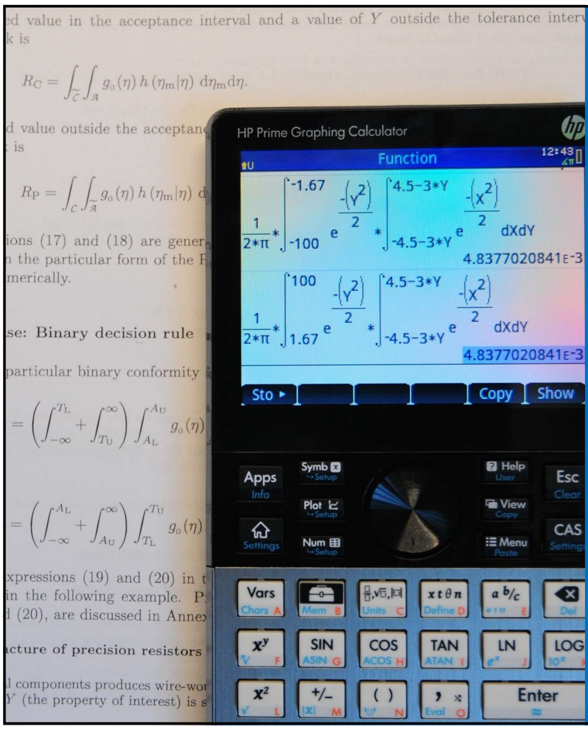
- The following approach is recommended :
- Case 1 : “**compliance**” is reported
- Case 4 : “**non-compliance**” is reported
- Case 2 and 3 : “**Not possible to state compliance**”



From ILAC-G8:2009

ILAC-G8:03/2009 on Statement of Compliance

- The statement of compliance should not be confused with inspection or product certification. For this purpose the following remark can be added “*The measurement results and statement of compliance only relate to the unit under test*”
- The coverage probability for the expanded uncertainty should be specified.
- For calibration, measurement uncertainty should always be considered when making statement of compliance.
- For testing, some specifications and standard might not include measurement uncertainty. Special care should be taken in the reporting.



ILAC-G8:09/2019 Guidelines on Decision rules and Statements of Conformity

Revision of ILAC-G8:03/2009 to ILAC-G8:09/2019

- From ISO/IEC 17025:2005 to ISO/IEC 17025:2017
- From "**Reporting of compliance with specification**" to "**Decision Rules and Statements of Conformity**".
- Introduction of "**Decision Rule**"
- A complete re-writing of ILAC-G8:03/2009
- To facilitate the change in the concept of stating compliance or conformity to a specification/requirement for assessors, laboratories and users of laboratory services.

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Conformity and Compliance

Conformity assessment	Activity to determine whether specified requirements relating to a product, process, system, person or body are fulfilled. (ISO/IEC 17000:2004)
Compliance	Not defined in ISO/IEC 17000 and VIM Affirmative indication or judgment that a product has met the requirements of the relevant specifications, contract or regulation, also the state of meeting the requirements. (Early version of TL 9000)

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ILAC-G8:09/2019 Preamble

The revised ISO/IEC 17025:2017 recognizes that no single decision rule can address all statements of conformity across the diverse scope of testing and calibration.

This document provides:

a) overall guidance on how to select appropriate decision rules; and

b) guidance on compiling the required elements of a decision rule if no standard published rules apply.

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From ILAC-G8:09/2019

Definitions

Tolerance Limit	specified upper or lower bound of permissible values of a property
Tolerance interval	interval of permissible values of a property
Acceptance limit	specified upper or lower bound of permissible measured quantity values
Acceptance interval	interval of permissible measured quantity values
Guard band	interval between a tolerance limit and a corresponding acceptance limit

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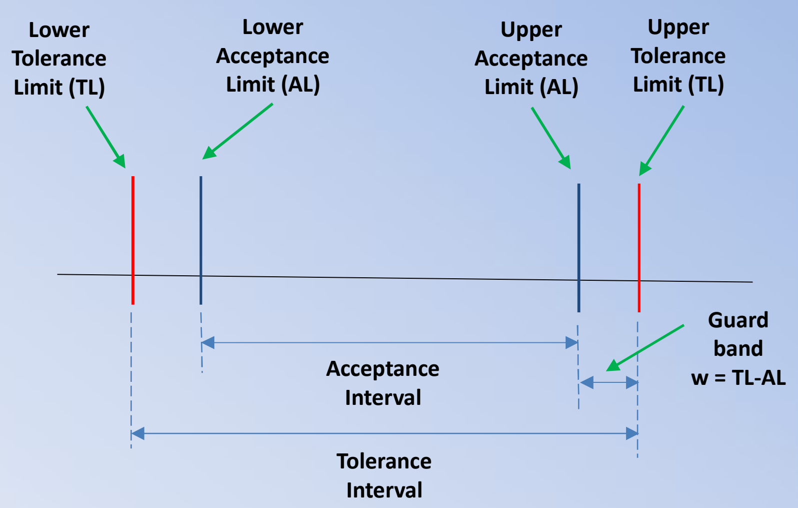
From ILAC-G8:09/2019

Definitions



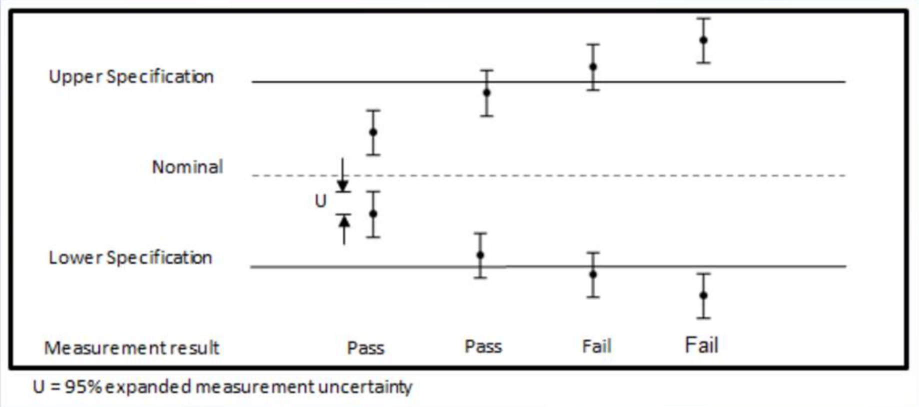
Decision Rule

Decision rule	rule that describes how measurement uncertainty is accounted for when stating conformity with a specified requirement.

- A measurement may result in a decision on conformity (acceptance) using one guard band and rejection if a larger guard band is used.
- Conformity with a requirement depends on the decision rule employed.
- The decision rule should be agreed before the measurements are taken.

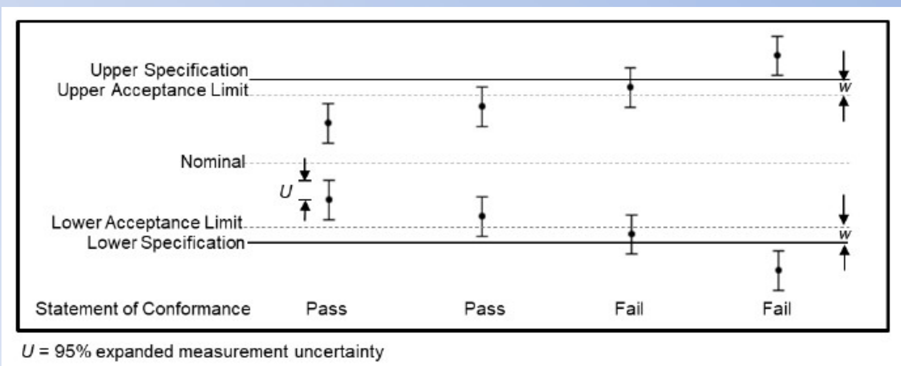
From ILAC-G8:09/2019

Example of Decision Rule
Binary statement for simple acceptance rule (w = 0)



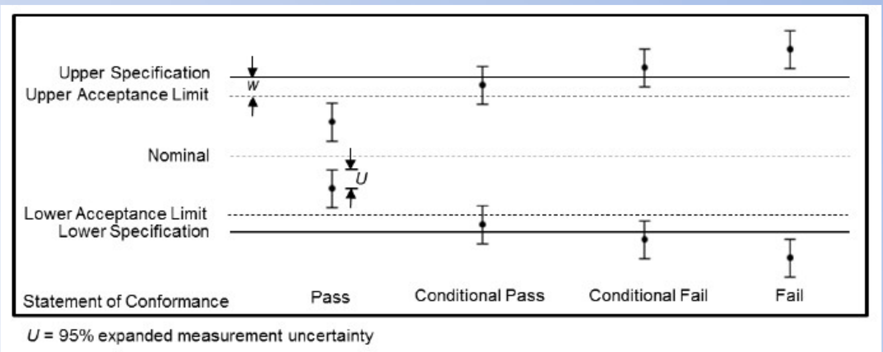
From ILAC-G8:09/2019

Example of Decision Rule
Binary statement with guard band



From ILAC-G8:09/2019

Example of Decision Rule
Non-Binary statement with guard band



From ILAC-G8:09/2019

Measurement Uncertainty indirectly taken into account

- In many standards, measurement uncertainty is taken indirectly into consideration. Guard bands are not used.
- Some examples are :
- OIML R76-1:2006 (NAWIs) cl. 3.7.1 "*…the standard masses used for the type examination or verification of an instrument …. shall not have an error greater than 1/3 of the MPE. If they belong to class E2 or better, their uncertainty is allowed to be not greater than 1/3 of the MPE of the instrument (the tolerance)*"
- OIML R117-1:2007 Dynamic measuring systems for liquids other than water Part 1: Metrological and technical requirements A.2 Uncertainties of measurement: "*When a test is conducted, the expanded uncertainty of the determination of errors on indications of volume or mass shall be less than one-fifth of the maximum permissible error (MPE) (the tolerance)*"

From ILAC-G8:09/2019

Measurement Uncertainty directly taken into account

- ISO/IEC 17025:2017 requires measurement uncertainty be taken into account when making statements of conformity.
- The approach varies significantly vary depending on the situation and different guard bands applied.
- Usually the guard band w is selected as rU (U = expanded measurement uncertainty U). For a binary decision rule, a measured value below the acceptance limit AL = TL – w is accepted.

Specific Risk and Global Risk

Specific risk	the probability that an accepted item is non-conforming, or that a rejected item does conform. This risk is based on measurements of **a single item**.
Specific consumer risk	the probability that a particular accepted item is non-conforming
Specific producer risk	the probability that a particular rejected item is conforming.

From ILAC-G8:09/2019

Specific Risk and Global Risk

Decision rule	Guard band w	Specific Risk
6 sigma	3 U	< 1 ppm PFA
3 sigma	1,5 U	< 0.16% PFA
ILAC G8:2009 rule	1 U	< 2.5% PFA
ISO 14253-1:2017 [5]	0,83 U	< 5% PFA
Simple acceptance	0	< 50% PFA
Uncritical	-U	Item rejected for measured value greater than $AL = TL + U$ < 2.5% PFR
Customer defined	r U	Customers may define arbitrary multiple of r to have applied as guard band.

Table 1. PFA – Probability of False Accept and PFR – Probability of False Reject
(Assumes a single sided specification and normal distribution of measurement results)

From ILAC-G8:09/2019

Specific Risk and Global Risk

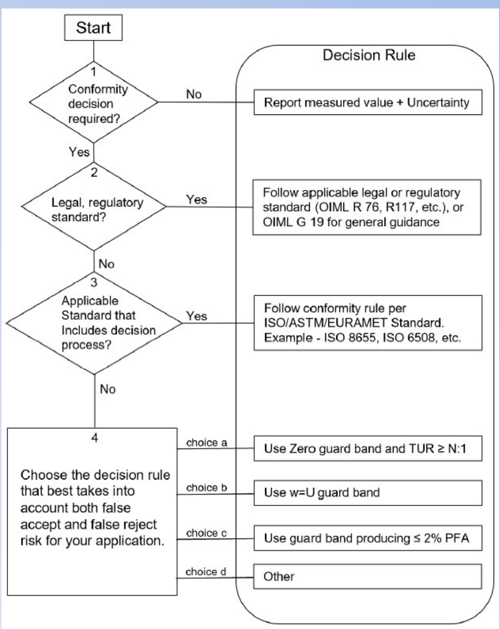
Global risk	is the average probability that an accepted item is non-conforming, or that a rejected item does conform. It does not directly address the probability of false accept to any single item, discrete measurement result or individual workpiece
Global consumer risk	the probability that a non-conforming item will be accepted based on a future measurement result
Global producer risk	the probability that a conforming item will be rejected based on a future measurement result

From ILAC-G8:09/2019

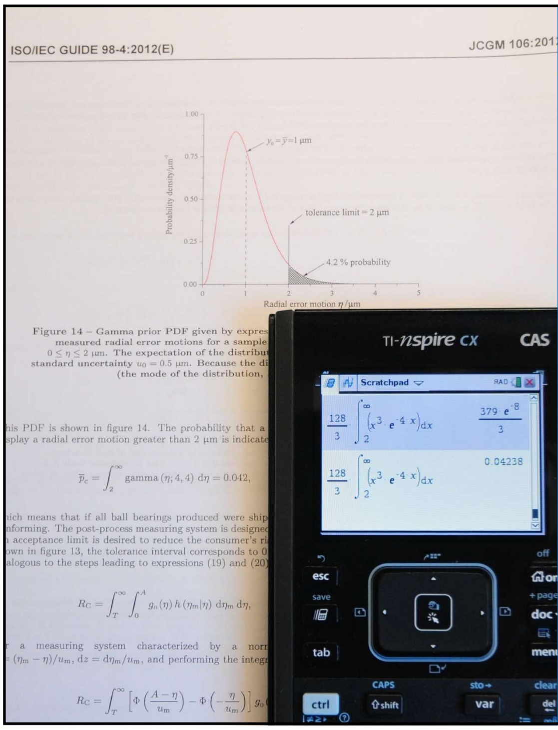
Selection of Decision Rule

- Where choices of decision rules are available, customers and laboratories will need to discuss levels of risk regarding the probability of false acceptance and false rejects associated with available decision rules.
- There is no one single decision rule that covers all testing and calibration cases

Selection of Decision Rule



From ILAC-G8:09/2019

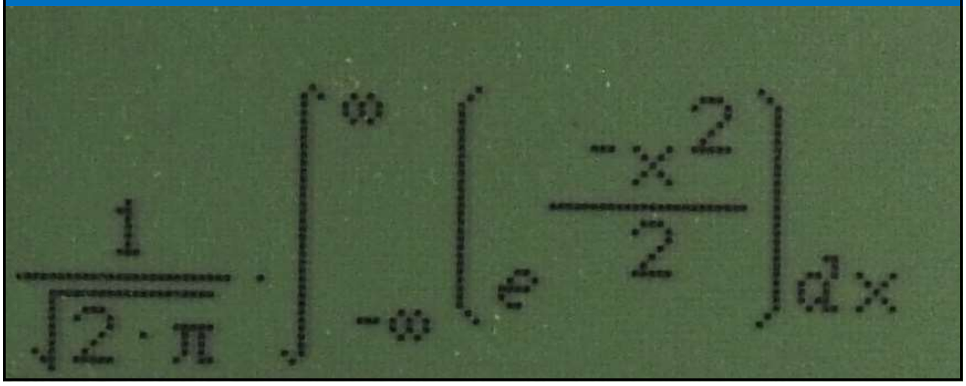


ISO/IEC Guide 98-4 JCGM 106:2012 Evaluation of measurement data – The role of measurement uncertainty in conformity assessment

Scope

- Provides guidance and procedures for assessing the conformity of an item with specified requirements
  - Item has a **measurable property**
  - Interval of permissible value specified (**tolerance interval**)
- The procedures can be used to realize an **acceptance interval**.
- Acceptance limits can be chosen so as **to balance the risks** associated with accepting non-conforming items (consumer's risk) or rejecting conforming items (producer's risk).
- Two types of conformity assessment problems are addressed.
  - Setting of acceptance limits so that a desired conformance probability for a **single measured item** is achieved.
  - Setting of acceptance limits to assure an acceptable level of confidence **on average as a number of items** are measured.

Specific Risk
for a single measured item



Probability of conformity with specified requirement

- In many cases it is reasonable to characterise measuand Y by a normal distribution with estimate y and standard uncertainty u. The probability density function (pdf) of Y is given by
- $g(\eta) = \frac{1}{u\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-y}{u}\right)^2}$
- The probability that Y lies in the interval [a,b]
- $\Pr(a \leq Y \leq b) = \Phi\left(\frac{b-y}{u}\right) - \Phi\left(\frac{a-y}{u}\right)$
- $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2} dt$ (standard normal distribution function)

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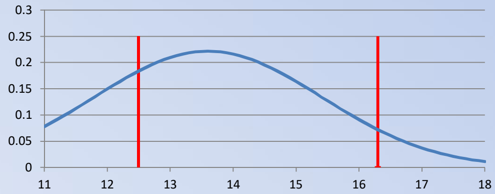
Probability of conformity with specified requirement

- Example : The tolerance interval for the kinematic viscosity Y of a sample of motor oil is [12.5 ,16,3] mm^2s^{-1}. A sample is measured to have estimate y = 13.6 mm^2s^{-1} with standard uncertainty u = 1.8 mm^2s^{-1}. What is the conformance probability ?

- The probability that Y lies in the interval [a,b]
- $\Pr(12.5 \le Y \le 16.3) = \Phi\left(\frac{16.3-13.6}{1.8}\right) - \Phi\left(\frac{12.5-13.6}{1.8}\right)$
- =0.93-0.27 = 0.66

- The probability that the sample of oil conforms to specification is 66%



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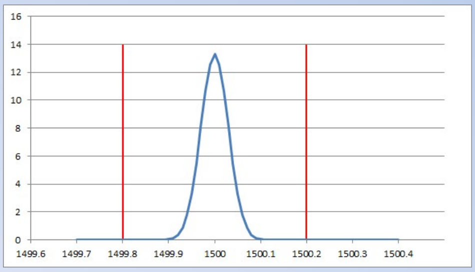
Specific Consumer's Risk based on simple acceptance (no guard band)

- Example : The tolerance interval for the resistance Y of a sample of resistor is [1499.8, 1500.2] ohm. A sample is measured to have estimate y = 1500 ohm with standard uncertainty u = 0.03 ohm. What is the specific consumer's risk ?
- The probability that Y lies in the interval [a,b]
- $\Pr(1499.8 \le Y \le 1500.2) = \Phi\left(\frac{1500.2-1500}{0.03}\right) - \Phi\left(\frac{1499.8-1500}{0.03}\right)$
- =1 - 0 = 1
- The probability that the resistor sample conforms to specification is 100%
- The specific consumer' risk = 1 – 1 = 0 %



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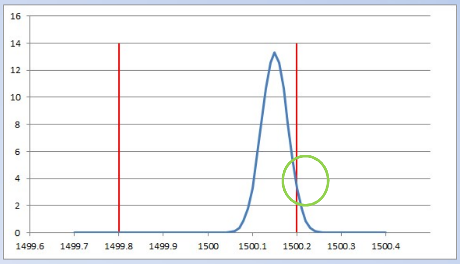
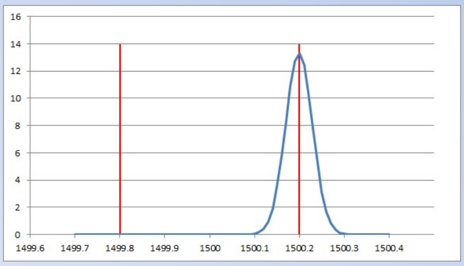
Specific Consumer's Risk when the measured value is close to the tolerance limit

- Example : The tolerance interval for the resistance Y of a sample of resistor is [1499.8, 1500.2] ohm. A sample is measured to have estimate y = 1500.15 ohm with standard uncertainty u = 0.03 ohm. What is the specific consumer's risk ?
- The probability that Y lies in the interval [a,b]
- $\Pr(1499.8 \le Y \le 1500.2) = \Phi\left(\frac{1500.2-150\ \ .15}{0.03}\right) - \Phi\left(\frac{1499.8-1500.15}{0.03}\right)$
- =0.952 - 0 = 0.952
- The probability that the resistor sample conforms to specification is 95.2%
- The specific consumer' risk = 1 – 0.952 = 4.8 %



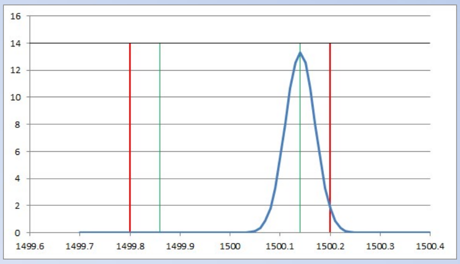
Specific Consumer's Risk when the measured value is very close to the tolerance limit

- Example : The tolerance interval for the resistance Y of a sample of resistor is [1499.8, 1500.2] ohm. A sample is measured to have estimate y = 1500.199 ohm with standard uncertainty u = 0.03 ohm. What is the specific consumer's risk ?
- The probability that Y lies in the interval [a,b]
- $\Pr(1499.8 \le Y \le 1500.2) = \Phi\left(\frac{1500.2-1500.199}{0.03}\right) - \Phi\left(\frac{1499.8-150\ \ .199}{0.03}\right)$
- =0.513 - 0 = 0.513
- The probability that the resistor sample conforms to specification is 51.3%
- The specific consumer' risk = 1 – 0.513 = 48.7 % !



Specific Consumer's Risk with Guard Band (w = U)

- Example : The tolerance interval for the resistance Y of a sample of resistor is [1499.8, 1500.2] ohm. A sample is measured to have estimate y = 1500.14 ohm with standard uncertainty u = 0.03 ohm. What is the specific consumer's risk if guard band w = U is used ?
- The acceptance interval become [1499.86, 1500.14]
- $\Pr(1499.8 \leq Y \leq 1500.2) = \Phi\left(\frac{1500.2-150\ \ .14}{0.03}\right) - \Phi\left(\frac{1499.8-1500.14}{0.03}\right)$
- =0.977 - 0 = 0.977
- The probability that the resistor sample conforms to specification is 97.7%
- The worst case specific consumer' risk = 1 – 0.977 = 2.3 %



Specific Producer's Risk
based on simple acceptance (no guard band)

- Example : The tolerance interval for the resistance Y of a sample of resistor is [1499.8, 1500.2] ohm. A sample is measured to have estimate y = 1500.25 ohm with standard uncertainty u = 0.03 ohm. What is the specific producer's risk ?
- The probability that Y lies in the interval [a,b]
- $\Pr(1499.8 \leq Y \leq 1500.2) = \Phi\left(\frac{1500.25-1500.2}{0.03}\right)$
- =0.048
- The probability that the resistor sample conforms to specification is 4.8%. Therefore, the specific producer's risk = 4.8 %

Global Risk

on average as
a number of items are measured

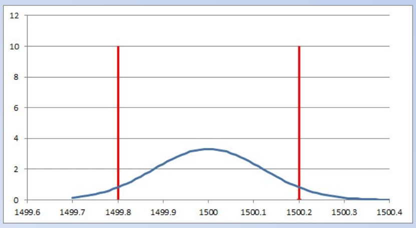
Need to consider Probability Distribution Function (PDF) of both production process and measurement system

$$\frac{1}{2*\pi} * \int_{-100}^{-1.67} e^{\frac{-\left(Y^2\right)}{2}} * \int_{-4.5-3*Y}^{4.5-3*Y} e^{\frac{-\left(X^2\right)}{2}} dXdY$$

4.8377020841E-3

Example of Global Consumer's Risk Production Process

- The tolerance interval for the resistance Y of a sample of resistor is [1499.8, 1500.2] ohm.
- A machine can produce the resistors with nominal resistance value Y of 1500 ohm with standard deviation of 0.12 ohm. The distribution is assumed to be normal.
- For a typical resistor produced by the machine, the conformance probability is
- $\Pr(1499.8 \le Y \le 1500.2) = \Phi\left(\frac{1500.2-1500}{0.12}\right) - \Phi\left(\frac{1499.8-15}{0.12}\right)$
- =0.952 – 0.048 = 0.904
- The probability that the resistor produced conforms to specification is 90.4%



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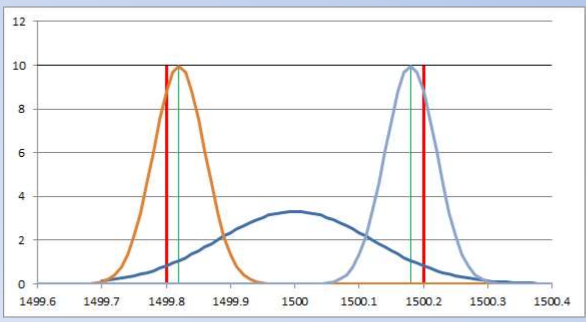
Example of Global Consumer's Risk Measurement System

- A calibrated ohmmeter can measure the value of the resistor In production with standard uncertainty of 0.04 ohm.
- If we adopt a guard band of 0.25 U, the acceptance interval become [1499.82, 1500.18] ohm. What will be the global consumer's risk ?



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Joint PDF of Production Process and Measurement System

- The joint probability density can be written as a product of densities.

The probability that the measurand Y is outside the tolerance interval and measured value Y_m is within the acceptance interval	=	the probability that the **production process** produces an item outside the tolerance interval	x	the probability that the **measuring system** produces Y_m within the acceptance interval, given that the measurand Y is outside the tolerance interval

$$f(\eta, \eta_m) = g_0(\eta) \times h(\eta_m \mid \eta)$$

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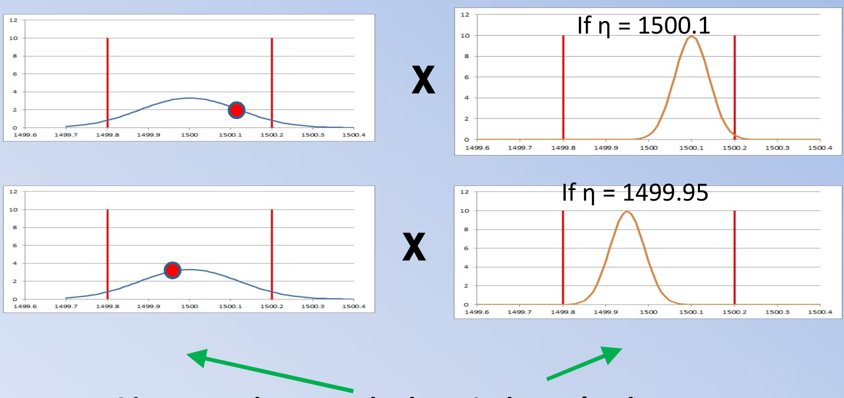
Joint PDF of Production Process and Measurement System

- The joint probability density can be written as a product of densities.



$$f(\eta, \eta_m) = g_0(\eta) \times h(\eta_m | \eta)$$

Global Consumer's Risk

- For a measured value in the acceptance interval and a value of Y outside the tolerance interval, the global consumer's risk is

$$R_C = \int_{\tilde{C}} \int_{\mathcal{A}} g_0(\eta)\, h(\eta_m|\eta)\, d\eta_m d\eta$$

$$g(\eta) = \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-1500}{0.12}\right)^2}$$

$$h(\eta_m|\eta) = \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta_m-\eta}{0.04}\right)^2}$$

A = [1499.82, 1500.18]
C = [1499.80, 1500.20]

Global Consumer's Risk

$$R_C = \int_{\tilde{C}} \int_{\mathcal{A}} g_o(\eta)\, h(\eta_m|\eta)\, d\eta_m d\eta$$

$$R_C = \int_{-\infty}^{1499.80} \int_{1499.82}^{1500.18} \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-150}{0.12}\right)^2} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta m-\eta}{0.04}\right)^2} d\,\eta_m d\eta \;+$$
$$\int_{1500.2}^{\infty} \int_{1499.82}^{1500.18} \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-1500}{0.12}\right)^2} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta m-\eta}{0.04}\right)^2} d\,\eta_m d\eta$$

the above is simplified to

$$R_C = \int_{-\infty}^{1499.80} \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-1500}{0.12}\right)^2} \left(\int_{1499.82}^{1500.18} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta m-\eta}{0.04}\right)^2} d\,\eta_{m)}\right) d\eta \;+$$
$$\int_{1500.2}^{\infty} \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-1500}{0.12}\right)^2} \left(\int_{1499.82}^{1500.18} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta m-\eta}{0.04}\right)^2} d\,\eta_m\right) d\eta$$

Global Consumer's Risk

$$R_C = \int_{\tilde{C}} \int_{\mathcal{A}} g_o(\eta)\, h(\eta_m|\eta)\, d\eta_m d\eta$$

$$R_C = \int_{-\infty}^{1499.80} \int_{1499.82}^{1500.18} \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-1500}{0.12}\right)^2} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta m-\eta}{0.04}\right)^2} d\,\eta_m d\eta \;+$$
$$\int_{1500.2}^{\infty} \int_{149\ .82}^{1500.18} \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-1500}{0.12}\right)^2} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta m-\eta}{0.04}\right)^2} d\,\eta_m d\eta$$

the above is simplified to

$$R_C = \int_{-\infty}^{1499.80} \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-1500}{0.12}\right)^2} \left(\int_{149\ .82}^{1500.18} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta m-\eta}{0.04}\right)^2} d\,\eta_{m)}\right) d\eta \;+$$
$$\int_{1500.2}^{\infty} \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-1500}{0.12}\right)^2} \left(\int_{149\ .82}^{1500.18} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta m-\eta}{0.04}\right)^2} d\,\eta_m\right) d\eta$$

Don't worry about the detailed calculations that follow. They only give you an idea how the results of example 9.5.3.2 in JCGM 106 are obtained

Global Consumer's Risk

Let y = (η-1500)/0.12, the above is further simplified to

$$R_C = \int_{-\infty}^{-1.667} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \left(\int_{1499.82}^{1500.18} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta_m - 0.12y - 1500}{0.04}\right)^2} d\,\eta_m \right) d \; + \int_{1.667}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \left(\int_{1499.82}^{1500.18} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta_m - 0.12y - 1500}{0.04}\right)^2} d\,\eta_m \right) dy$$

Let x = (η_m-0.12y-1500)/0.04, we get

$$R_C = \int_{-\infty}^{-1.667} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \left(\int_{-4.5-3y}^{4.5-3y} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} d\,x \right) dy \; + \int_{1.667}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \left(\int_{-4.5-3y}^{4.5-3y} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} d\,x \right) dy$$

Global Consumer's Risk

Let y = (η-1500)/0.12, the above is further simplified to

$$R_C = \int_{-\infty}^{-1.667} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \left(\int_{1499.82}^{1500.18} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta_m - 0.12y - 1500}{0.04}\right)^2} d\,\eta_m \right) d \; + \int_{1.667}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \left(\int_{1499.82}^{1500.18} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta_m - 0.12y - 1500}{0.04}\right)^2} d\,\eta_m \right) dy$$

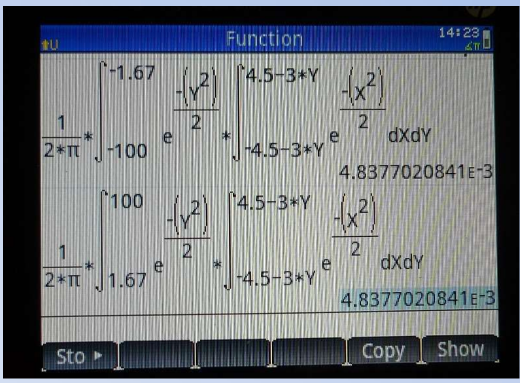
Let x = (η_m-0.12y-1500)/0.04, we get

$$R_C = \frac{1}{2\pi} \int_{-\infty}^{-1.667} e^{-\frac{1}{2}y^2} \left(\int_{-4.5-3y}^{4.5-3y} e^{-\frac{1}{2}x^2} d\,x \right) dy \; + \frac{1}{2\pi} \int_{1.667}^{\infty} e^{-\frac{1}{2}y^2} \left(\int_{-4.5-3y}^{4.5-3y} e^{-\frac{1}{2}x^2} d\,x \right) dy$$

This integral can be evaluated numerically.

Global Consumer's Risk

$$R_C = \frac{1}{2\pi}\int_{-\infty}^{-1.667} e^{-\frac{1}{2}y^2}\left(\int_{-4.5-3y}^{4.5-3y} e^{-\frac{1}{2}x^2}\,d\,x\right)dy + \frac{1}{2\pi}\int_{1.667}^{\infty} e^{-\frac{1}{2}y^2}\left(\int_{-4.5-3y}^{4.5-3y} e^{-\frac{1}{2}x^2}\,d\,x\right)dy$$



R_c = 0.00484+0.00484=0.00968

The Global consumer's risk is reduced to 0.97%

Global Producer's Risk

$$R_P = \int_C \int_{\tilde{A}} g_0(\eta)\, h\left(\eta_m|\eta\right)\, d\eta_m d\eta.$$

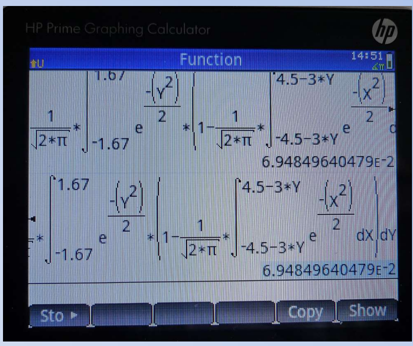
$$R_p = \int_{1499.8}^{1500.2}\int_{-\infty}^{1499.82} \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-1500}{0.12}\right)^2} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta_m-\eta}{0.04}\right)^2} d\,\eta_m d\eta + \int_{1499.8}^{1500.2}\int_{1500.18}^{\infty} \frac{1}{0.12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta-150}{0.12}\right)^2} \frac{1}{0.04\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\eta_m-\eta}{0.04}\right)^2} d\,\eta_m d\eta$$

the above is simplified to

$$R_p = \frac{1}{\sqrt{2\pi}}\int_{-1.667}^{1.667} e^{-\frac{1}{2}y^2}\left(1 - \frac{1}{\sqrt{2\pi}}\left(\int_{-4.5-3y}^{4.5-3y} e^{-\frac{1}{2}x^2}\,d\,x\right)dy\right.$$

Global Producer's Risk

$$R_p = \frac{1}{\sqrt{2\pi}} \int_{-1.667}^{1.667} e^{-\frac{1}{2}y^2} (1 - \frac{1}{\sqrt{2\pi}} (\int_{-4.5-3y}^{4.5-3y} e^{-\frac{1}{2}x^2} d\,x) dy$$



R_p = 0.0695

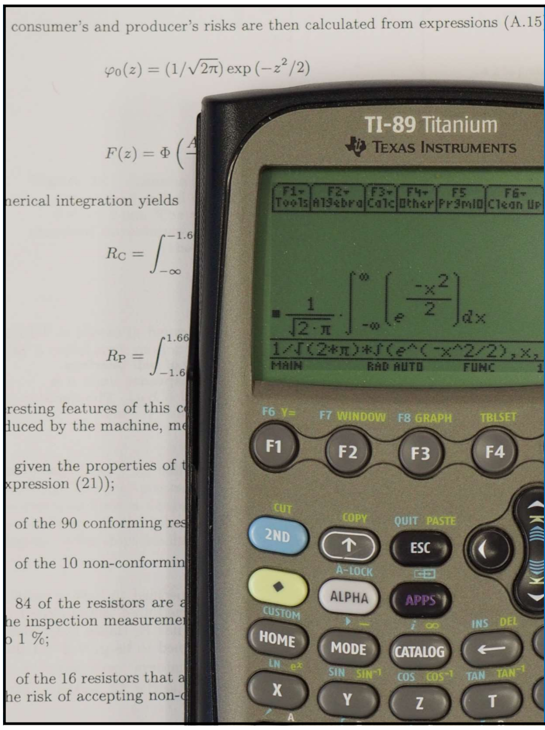
The Global producer's risk is 6.95%

Example of Global Risks

- The tolerance interval for a sample of resistor is [1499.8, 1500.2] ohm.
- A machine can produce the resistors with nominal 1500 ohm with standard deviation of 0.12 ohm.
- A calibrated ohmmeter has a standard uncertainty of 0.04 ohm.
- If the acceptance interval is [1499.82, 1500.18] ohm. What will be the global risks ?

Results

- For a sample of 100 resistors, 90 conform and 10 do not conform.
- Of the 90 conform, 83 are accepted and 7 falsely rejected as non-conforming
- Of the 10 non-conforming resistors, 9 are rejected and one falsely accepted as conforming
- 84 resistors are accepted, 83/84 ~ 99% conform, 1 % out of tolerance.
- 16 resistors rejected, 7/16 ~ 44% actually conform with specification.



Thank you

謝謝